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PROBABILISTIC COLLOCATION FOR EFFICIENT UNCERTANTY ANALYSIS IN GROUNDWATER FLOW

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Summary. Let Ω be a bounded domain in $\mathbb{R}^2$ with boundary Γ. We consider the following stochastic groundwater flow problem,

$$\begin{cases} \nabla \cdot \left(-\alpha\left(\mathbf{x},\omega\right)\nabla u\right) = f\left(\mathbf{x}\right), & \mathbf{x}\in\Omega \\ u\left(\mathbf{x},\omega\right) = g\left(\mathbf{x}\right), & \mathbf{x}\in\Gamma \end{cases} \tag{1}$$

where $\alpha(\mathbf{x},\omega)$ denotes the heterogeneous stochastic permeability field. We set $\alpha\left(\mathbf{x},\omega\right) = e^{k(\mathbf{x},\omega)}$ and assumed that $k(\mathbf{x},\omega)$ is normally distributed with mean $\bar{k}\left(\mathbf{x}\right)$ and covariance $C_k(\mathbf{x},\mathbf{x}')$. To solve the stochastic problem (1), we combine a low order mixed finite element discretization in physical space with an optimal collocation technique in probability space for uncertainty propagation. The probabilistic collocation allows building the polynomial chaos expansions of the model response stochastic fields (pressure and variance). In presence of correlated random fields, the Karhunen-Loève expansion is an efficient method to reduce, a priori, the infinite-dimensional probability space to a finite-dimensional one defined by the first eigenmodes of the covariance function. We show that a reduction of the eigenmodes is possible, a posteriori, by *screening* the most relevant ones by means of variance-based sensitivity indices. The latter are easily computed with the polynomial chaos coefficients. The stochastic error analysis is investigated through numerical experiments. It is demonstrated that the strategy leads to significant reduction of the computational cost with good accuracy.