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HIGH-ORDER DISCONTINUOUS GALERKIN FOR UNSTEADY PROBLEMS IN HIGHLY ABSORVENT MEDIA

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Summary. Homogeneous Surface Diffusion Model (HSDM) is widely used for modeling adsorption from aqueous dissolutions with highly absorbent media. In [1] a detailed dimesnionless analysis is performed, and it is shown limit behaviour in terms of the model parameters. Accurate modeling of these problems requires the ability to handle sharp fronts and shocks when dealing with non-linear isotherm equilibrium relationships. Therefore, obtaining an accurate solution represents a challenging numerical problem.

Because of the difficulty associated with obtaining an accurate solution which is able to represent all the different profiles of the model solution, DG methods have received increasing attention in computational fluid dynamics, specially becaue they conserve mass locally and they are well suited for *hp* adaption, among many others advantages. However, in the presence of strong shocks DG methods also need a stabilization procedure, usually introduced by slope limiting procedures [2]. Despite these non-linear operators avoid spurious oscillations and provide a stable solution, they have two main drawbacks: they restrict the time integration to a special class of explicit Runge-Kutta schemes and they reduce the order of the approximation to a linear or even constant one, introducing an excessive amount of dissipation and therefore, requiring mesh adaptivity in order to obtain an accurate solution.

In this work the artificial diffusion discontinuous Galerkin method presented in [3] is applied. The method does not depend on the formulation and it allows to solve the original problem without any simplification neither *h*-refinement, maintaining both computational efficiency and solution accuracy. An accurate solution free of spurious oscillations is obtained and the shock profile is resolved as it advances in time without any phase error.

Numerical results suggest that the artificial diffusion DG method can treat HSDM problems over a wide range of modeling parameters, including both, advection- and reaction-dominated problems. In addition, it is shown that the proposed method can sharply capture local phenomena with high-order accuracy without mesh adapatation.

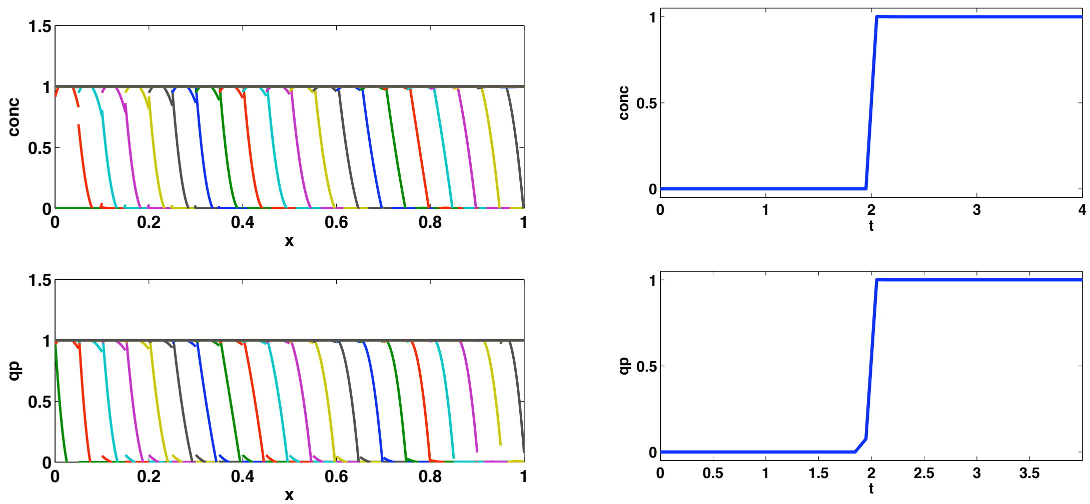

Figure 1: Simulation with the artificial diffusion DG method on a mesh of 10 elements with degree of interpolation $p = 2$. Left figure: evolution of the concentration in porous media and of the mean value of adsorbed mass in the particles. Right figure: breaktrough curves for both variables.

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