XVIII International Conference on Water Resources
CMWR 2010
J. Carrera (Ed)
CIMNE, Barcelona 2010

SPATIALLY ADAPTIVE FINITE ELEMENTS WITH TEMPORAL ADAPTION USING LOCAL TRUNCATION ERROR CONTROL FOR VARIABLY SATURATED FLOW

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Summary. Variably saturated groundwater flow problems are often modeled with Richards' equation, a nonlinear partial differential equation. Obtaining robust numerical solutions efficiently continues to be challenging, particularly for infiltration into non-uniform porous media. This is due in part to the constitutive relations required to close the system which can be non-Lipschitz continuous and non-differentiable at the water table. This work is motivated by problems involving a sharp wetting front that propagates through the domain, requiring a sufficiently small time step while the front is forming and a significant number of nodes to capture the location of the front. Previous work has focused on spatially adaptive finite element methods with heuristic time stepping and more recent efforts have addressed the mixed-form of Richards' equation using adaptive finite difference schemes with variable order temporal integration to improve efficiency.

In this work, we present an *h*-adaptive Galerkin finite element approach that coarsens and refines the mesh based on an a priori finite element error indicator paired with a temporal adaption scheme that controls the location truncation error at each time step. The temporal adaption scheme is based on linear extrapolation for smooth functions, but has been rigorously proven to work for non-smooth infiltration problems when a finite-difference Jacobian is used to find the Newton-step.

We present numerical results for the pressure-head form of Richards' equation for infiltration problems into silt and clay. We provide error and work measures to demonstrate the performance of the joint spatial-temporal adaption algorithm when compared to a fixed grid approach with temporal error control, a fixed grid approach with heuristic temporal adaption, and a spatially adaptive approach with heuristic temporal adaption. We show that the joint scheme results in less nonlinear solver failures, larger time steps, temporal error bounded by a user specified tolerance, and accurate results with fewer elements.