XVIII International Conference on Water Resources
CMWR 2010
J. Carrera (Ed)
CIMNE, Barcelona 2010

VARIATIONAL MULTISCALE METHODS FOR NONLINEAR PROBLEMS

Jan Ole Skogestad[*], Jan M. Nordbotten[†]

[*] Department of Mathematics, University of Bergen
Johannes Bruns gate 12, 5008 Bergen, Norway
e-mail: jan.skogestad@math.uib.no, web page: http://math.uib.no

[†] Department of Mathematics, University of Bergen
Johannes Bruns gate 12, 5008 Bergen, Norway
e-mail: jan.nordbotten@math.uib.no, web page: http://math.uib.no

Summary. Modelling of flow in porous media is important for various applications such as oil recovery, CO_2 storage, geothermal energy and ground water resources. In this work we focus on solving the equation governing the pressure in a compositional model for multiphase flow in porous media. This equation is typically nonlinear and near-elliptic.

Large variations on small scales that affect the overall solution has lead to an interest in upscaling and multiscale methods. The latter has especially gained interest in the last few years, and integrates the upscaling part into the numerical solution method. A typical multiscale approach is to first linearize the equation, and then apply the multiscale method. Another path, which we follow here, is to apply multiscale methods directly on the nonlinear problem. This is a more challenging approach, and it is still unclear how it performs in comparison with more standard methods.

The particular class of multiscale methods we consider here is the variational multiscale methods (VMS). They are built on the variational formulation of the equations, and split the problem into a coarse scale and a fine scale problem. For linear problems, VMS is well understood. For nonlinear problems less is clear about the performance of the method since fine scale problems are now coupled to the coarse scale problem. We propose to solve this coupled problem by a subspace iteration technique.

VMS has been shown to be closely connected to domain decomposition methods, a field which during the past couple of decades has provided many useful tools for solving partial differential equations. The nonlinear extension of domain decomposition preconditioned iterative methods is additive Schwarz preconditioned inexact Newton (ASPIN). Thus, this forms the natural benchmark for nonlinear VMS to be compared against. In this work, we study the applicability of VMS and ASPIN to problems arising in flow in porous media.