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On the use of strong equilibrium for obtaining error bounds and implementing adaptivity in FE computations

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ABSTRACT

Solutions of complementary nature (one being compatible, the other equilibrated, so that their errors are orthogonal) can be used to assess the quality of approximate solutions of problems in solid mechanics. Although this property was recognised and used to obtain solution bounds almost one century ago[1], its application in finite element practice is still very limited.

Most users consider that finite element models have to be based on a (more or less) compatible approximation of the displacements, which leads to a weak form of equilibrium, which is thought to be strong by some. The existence of formulations based on the approximations of the stress field, which lead to strong forms of equilibrium and weak forms of compatibility is often ignored.

Element by element recovery techniques may also be used to obtain these equilibrated solutions, which together with the compatible ones leads to the bounds of the solution error [2]. Their application is not always straightforward and the non-optimal nature of the equilibrated solutions obtained leads in some cases to very coarse bounds.

The research conducted by the authors in the last twenty-five years, in the development of equilibrium based finite element formulations and in their application within the context of error estimation and adaptivity, led to the recent publication of a book on these topics [3].

In this communication we will summarise some results presented in this book, together with a discussion of the points to consider when implementing finite element codes that embody a strong form of equilibrium.

REFERENCES

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