Solution of low Mach number aeracoustic flows using a Variational Multi-Scale formulation of the compressible Navier-Stokes equations written in primitive variables

Camilo Bayona*, **Joan Baiges and Ramon Codina**

International Center for Numerical Methods in Engineering (CIMNE)
Universidad Politécnica de Cataluña
Campus Norte UPC, 08034 Barcelona, Spain
*e-mail: cbayona@cimne.upc.edu

ABSTRACT

In this work we solve the compressible Navier-Stokes equations in order to directly simulate sound. We present a new formulation to solve the low Mach number limit by using the well known Navier-Stokes compressible equations written in primitive variables. The Variational Multi-Scale method is used to stabilize the finite element discretization, by including the orthogonal [1] and dynamic [2] subscales, and an implicit scheme for advancing in time.

Three essential features define the proposed aeroacoustic solver: the splitting of the pressure and temperature variables into a relative and a reference part, the definition of the matrix of stabilization parameters in terms of a modified velocity that accounts for the local compressibility, and the approximation of the dynamic stabilization matrix for the transient subscales. We also explain the numerical methods that we have implemented in order to overcome the challenges that arise in the aeroacoustic simulations, such as the Poinsot and Lelef non-reflecting boundary conditions [3].

The order of accuracy of the method is verified for two and three dimensional linear and quadratic elements using steady manufactured solutions. Acoustic waves have been observed for the flow past a cylinder problem. It has also been found that including the dynamic and the orthogonal subscales improves the accuracy of the variational method.

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