

Technique to find 10-order Symmetric Composition Methods of Symmetric Integrators

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ABSTRACT

Composition methods are useful when solving Ordinary Differential Equations (ODEs) as they increase the order of accuracy of a given basic numerical integration scheme. We will focus on symmetric composition methods involving some basic second order symmetric integrator with different step sizes. The introduction of symmetries into these methods simplifies the order conditions and reduces the number of unknowns. Several authors have worked in the search of the coefficients of these type of methods: the best method of order 8 has 17 stages [1], methods of order 8 and 15 stages were given in [2, 4, 5], 10-order methods of 31, 33 and 35 stages have been also found [1, 3]. In this work a technique that we have built to obtain n -order and s -stages methods is explained. Some results of the application of this technique to the search of 10-order symmetric composition methods of symmetric integrators of $s = 31$ stages (16 order conditions) are explained. Given some starting coefficients that satisfy the simplest five order conditions, the process followed to obtain the coefficients that satisfy the sixteen order conditions is provided.

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