A new CFD framework for transport equations: ENATE scheme for bidimensional problems

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ABSTRACT

1st and 2nd order schemes are widespread in CFD. For many industrial applications they are good enough, but in those where the accuracy is a key-factor, a discretization with higher accuracy than 2nd or 3rd order is required. A high-order approximation using few grid-nodes is the aim of ENATE scheme [1]-[2]. This accurate and robust numerical approach, that can get almost machine accuracy with relatively few nodes in 1D, uses the exact solution of an one-dimensional transport equation for working out the algebraic equation with integral coefficients. The current efforts are to extend it for multidimensional problems with the same accuracy as in 1D. This paper presents some efforts towards that goal in two-dimensional problems.

When dealing with a 2D Transport Equation some issues arise. The example shown below is that of an inviscid transport. As in [3], a pseudo-source that comes from the derivatives in the other coordinate direction appears in each direction. Two pseudo-ODEs are then obtained and added up to get the final discretized equation. The nodal equation using ENATE reads:

$$\begin{aligned}(\rho u + \rho v)_P\,\phi_P =& (\rho u)_W\phi_W + (\rho v)_S\phi_S \\ &+ IS_{WP} + IS_{SP} - \int_W^P \left(\frac{\partial J_y}{\partial y}\right)^{old} dx - \int_S^P \left(\frac{\partial J_x}{\partial x}\right)^{old} dy \end{aligned} \tag{1}$$

where ρu, ρv are an arbitrary convective terms, ϕ is a general variable, $J_x(=\rho u\phi)$ and $J_y(=\rho v\phi)$ are the total fluxes, and P, W, S, stand for current, west and south nodes. IS_{ab} and old refer to "Integral of the Source term from a to b" and "old iterate", respectively. In order to approximate accurately the red terms of Eqn.(1), special processing is required. Some numerical tests will be introduced and different ways to deal with these integrals such as Hermite interpolation, cubic splines and a new implicit numerical integration will be compared. Also, other treatments of 2D Transport Equation will be set forth.

REFERENCES

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