

Numerical experiments of preconditioners for Krylov subspace methods in linear systems with a regularly structured matrix

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ABSTRACT

The solutions to the Laplace equation are sought in the deformed sector domain with the Dirichlet boundary conditions. The sector domain is deformed by moving one vertex. The Laplace equation in generalised coordinates ξ, η becomes

$$-\left\{\left(\frac{\nabla^2 \xi}{J}\right)\phi\right\}_{\xi} - \left\{\left(\frac{\nabla^2 \eta}{J}\right)\phi\right\}_{\eta} + \left\{\left(\frac{\alpha}{J}\right)\phi\right\}_{\xi\xi} + \left\{\left(\frac{\beta}{J}\right)\phi\right\}_{\xi\eta} + \left\{\left(\frac{\gamma}{J}\right)\phi\right\}_{\eta\eta} = 0$$

The equation is discretized using three-point centered difference scheme. The resulting linear system $Ax = b$ is solved. The non-zero entries of A are $a_{k,k-I-1}, a_{k,k-I}, a_{k,k-I+1}, a_{k,k-1}, a_{k,k}, a_{k,k+1}, a_{k,k+I-1}, a_{k,k+I}, a_{k,k+I+1}$. The system is solved by Krylov subspace methods. Krylov subspace methods include BCG, CGS, BCGSTAB, BCGTAB2, GPBCG, BCGSafe, BCR, CRS, BCRSTAB, BCRSTAB2, GPBCR, BCRSafe, GMRES. They are right-preconditioned by the SGS, SSOR, ILU(0), MILU(0), ILU(1) [1], MILU(1) preconditioners. They are also compared with multigrid methods. Multigrain methods are geometric multigrid (Galerkin coarse grid approximation) with V-, W-, F-cycles and full multigrid. The CPU times are measured on a common workstation on a 768^2 grid. The result shows that the SSOR preconditioner is faster than ILU(0) whereas it is slower than MILU(0), MILU(1) and multigrid methods. The SSOR preconditioner is about equally efficient as the ILU(1) preconditioner in the convergence rate, but is much easier to program than ILU and MILU preconditioners and multigrid methods.

REFERENCES

- [1] Y. Saad, *Iterative methods for sparse linear systems*, SIAM, (2003).