

WENO-ADER finite volume numerical schemes: from homogeneous linear equations to non-linear systems of equations with source terms

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ABSTRACT

Finite volume numerical schemes are a very common choice for solving complex flows in the framework of computational fluid mechanics. Such methods have experienced a significant improvement over the past few decades due to the appearance of novel algorithms that ensure the preservation of high accuracy in both space and time when computing the solution. The ideas of ENO and WENO reconstructions [1] supposed a major step when seeking high order in space, however, high order in time was generally achieved by means of a Runge-Kutta time discretization, which proved to be inefficient due to the Butcher barrier. This issue was resolved by the introduction of the ADER approach [2], a high order generalization of Godunov's method. ADER schemes consist of two main steps: the first one, a high-order spatial reconstruction of the variables and the second one, the resolution of the so-called Derivative Riemann Problem (DRP), a high order extension of the Riemann problem (RP). In this work, we closely show the whole way for the design of ADER schemes with application to non-linear systems of equations with source terms departing from the definition of ADER schemes for homogeneous linear problems. We first present an efficient dimension-by-dimension WENO reconstruction and sub-cell derivative reconstruction that can be applied to an arbitrary number of dimensions as well as the procedure for the construction of ADER schemes for linear problems, which truly is of arbitrary order as the Cauchy-Kovalewski procedure can be implemented recursively. Based on such ideas, we then show how such methods can be extended to non-linear systems of conservation laws, focusing on the numerical treatment of the source terms when solving the DRP [3]. Moreover, midway problems, namely the linearized version of certain non-linear systems, will also be considered.

REFERENCES

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